DYNAMIC SIMULATION OF WHEELED VEHICLES: MODELS AND ALGORITHMS

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Abstract

Simulation of wheel-ground and vehicle-ground interactions is very important in many applications. Achieving accuracy and efficiency is challenging for both soft and hard terrains. This is not only because of the simulation and numerical challenges, but also due to the questionable nature of the terrain models. For example, the most widely used terramechanics model is not compatible with dynamic models and simulation. It is not a representative constitutive relation for a full range of dynamic conditions and applications. In general, the selection and development of the proper constitutive model and the parameterization of the ground properties are very challenging. We have been developing a unified framework for general wheel-ground interaction which can be used with whichever constitutive model and parameterization is selected and deemed to be representative for the particular application considered. The framework is based on a complementarity formulation and also uses the concept of kinematic constitutive relations, beside the other known concepts for modelling and parameterizing the soil properties. The framework makes it possible to consider the appropriate modelling of the terrain for a broad range of dynamic behaviours and simulation conditions. We will illustrate the material with several examples for off-road conditions.

Keywords: wheel-ground interaction, dynamic models and simulation, complementarity formulation

1. Introduction and Dynamics Modelling

The mechanics of a vehicle moving on some terrain can generally be represented with a finite degree of freedom model based on n generalized velocity components, collected in array $\mathbf{v}$, which describe the velocity field of the system, and the related configuration level parameterization using p generalized coordinates, given in array $\mathbf{q}$. In general, $p \geq n$. The general dynamics formulation for such a model can be given as

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{c} = \mathbf{f} + \mathbf{f}_{\mathrm{c}} + \mathbf{f}_{\mathrm{g}} \quad (1)$$

where $\mathbf{M}$ is the $n \times n$ mass matrix, $\mathbf{c}$ is the $n \times 1$ array of Coriolis and centrifugal terms, the $n \times 1$ array $\mathbf{f}_{\mathrm{c}}$ represents the generalized interaction forces among the elements of the vehicle model, $\mathbf{f}_{\mathrm{g}}$ contains the n generalized force components that represent the forces and moments transmitted from the ground to the vehicle through the wheel-ground contacts, and $\mathbf{f}$ includes the representation of any other force acting on the system elements.

A discrete-time formulation can also be derived from the above for the velocities considering a finite difference approximation for the accelerations. This can be accomplished using the first-order approximation

$$\dot{\mathbf{v}} = \frac{\mathbf{v}^{+} - \mathbf{v}}{h} \quad (2)$$

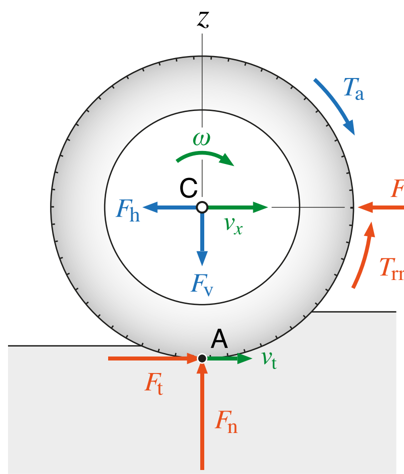

Figure 1: Wheel-terrain interaction forces

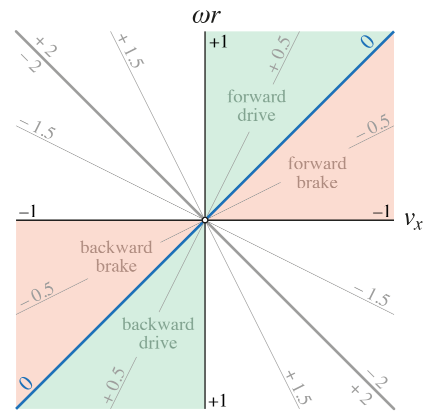

Figure 2: Slip ratio

where h is the time-step size. The superscript "+" denotes the unknown variables of the current step, and the variables without superscript are known, computed in the previous step. It can be shown that only such first-order approximations would work for more complex cases involving contacts and friction.

This above representation can correspond to different model forms depending on how the interactions among the vehicle model elements and related forces $\mathbf{f}_\mathrm{c}$ are represented. Commonly these interactions are represented using bilateral constraints, i.e., various types of joints, representing the structural topology of the vehicle. In this case, these constraints may already be implicitly embedded in the above model and the n generalized velocities associated with the model represent the minimum number of degrees of freedom with respect to these bilateral constraints. In that case, the related interaction forces are also eliminated and $\mathbf{f}_\mathrm{c} \equiv \mathbf{0}$. The other possible form for these structural constraints is that the related constraint forces are explicitly included and the constraints are added to the model with kinematic equations that can be compactly written as

$$\mathbf{B}\dot{\mathbf{v}} + \dot{\mathbf{B}}\mathbf{v} = \mathbf{0} \tag{3}$$

and $\mathbf{f}_\mathrm{c} = \mathbf{B}^\mathrm{T}\boldsymbol{\beta}$ where $\boldsymbol{\beta}$ represents the reaction forces/moments transmitted in the joints. These equations can also be discretized using Eq. (2). Even if these constraints and related reactions are considered explicitly in the formulation, there are various possible ways to handle them.

The more challenging aspect is the consideration of the interactions due to the wheel-ground contacts, represented by $\mathbf{f}_\mathrm{g}$. The resultant interaction force and moment components transmitted from the ground to each wheel can be represented by a 6×1 array $\boldsymbol{\lambda}_i$ with $i = 1, \ldots, s$, where s is the number of wheels. Based on this observation, the generalized force term can be further decomposed as $\mathbf{f}_\mathrm{g} = \mathbf{A}^\mathrm{T}\boldsymbol{\lambda}$ where $\boldsymbol{\lambda} = \left[\boldsymbol{\lambda}_1^\mathrm{T}, \ldots \boldsymbol{\lambda}_s^\mathrm{T}\right]^\mathrm{T}$, and $\mathbf{A}^\mathrm{T}$ represents the transformation from the resultant ground interaction forces and moments to the generalized force terms. At the same time $\mathbf{A}$ gives the transformation from the generalized velocity components to the Cartesian velocity components that are used to interpret each wheel-ground interaction, $\mathbf{u} = \left[\mathbf{u}_1^\mathrm{T}, \ldots \mathbf{u}_s^\mathrm{T}\right]^\mathrm{T}$, as $\mathbf{A}\mathbf{v} = \mathbf{u}$. Using this transformation, the power of the ground reaction forces can be expressed as $P_\mathrm{g} = \mathbf{u}^\mathrm{T}\boldsymbol{\lambda} = \mathbf{v}^\mathrm{T}\mathbf{f}_\mathrm{g}$.

For each individual wheel the ground reactions contained in $\boldsymbol{\lambda}_i$ need to be further analyzed and interpreted. If we use constitutive relations then those need to demonstrate and reflect the physical phenomena considered. With the notation of Fig. 1, expressions for F_c, F_t, F_n, and T_rr could be derived to represent the ground reaction forces directly affecting longitudinal motion, which are defined below. If we consider the coordinate directions given in Fig. 1, we can structure $\boldsymbol{\lambda}_i$ so that the first three components define the resultant force and the second three give the resultant moment about the selected reference point that is either the centre of the wheel or the contact point. In this case, the above forces would enter as $\lambda_i(1) = F_\mathrm{t} - F_\mathrm{c}$, $\lambda_i(3) = F_\mathrm{n}$, and $\lambda_i(5) = -T_\mathrm{rr} - F_\mathrm{t} r$, where r is the radius of the wheel. The other ground reaction components would come from the modelling of the lateral forces acting on the wheel.

The model for F_c, F_t, F_n, and T_rr can be derived based on the terramechanics representation selected. For example, the well-known pressure-sinkage relationship proposed by Bekker [3]

$$\sigma = \left(\frac{k_\mathrm{c}}{b} + k_\phi\right) z^n \tag{4}$$

and its modified versions play a central role, and form the basis of most terramechanics formulations and approaches. In this equation b is a geometric dimension depending on the experimental setup, k_c, k_ϕ, and n are parameters to be identified based on experimental tests, and z is the sinkage. However, a formula like this does not necessarily mean a valid constitutive relationship for all conditions. The physical meaning of the formula highly depends on the loading and experimental conditions and the material that is being characterized.

The representation of the terrain reactions is also highly dependent on the consideration of slip. A general definition of the slip ratio is [2, 3, 7]

$$i_\mathrm{s} = \frac{\omega r - v_x}{v_\mathrm{max}} \tag{5}$$

where v_x is the longitudinal velocity of the centre of the wheel, ω is the angular velocity about its axis, r is its radius, and

$$v_\mathrm{max} = \begin{cases} \omega r & \text{if} \quad |\omega r| \geqslant |v_x| \quad \text{(driving)} \\ v_x & \text{if} \quad |\omega r| < |v_x| \quad \text{(braking)} \end{cases} \tag{6}$$

The slip ratio $i_\mathrm{s} \in [0,1]$ is used for driving wheels, whereas $i_\mathrm{s} \in [-1,0)$ is used for braking or towed wheels (see Fig. 2). Nevertheless, a smoothing function can be used when velocities are very low, because the slip is not defined when the wheel is stopped. Therefore, the smooth slip can be defined as

$$i_\mathrm{s} = \frac{\omega r - v_x}{v_\mathrm{max}} \left(1 - \mathrm{e}^{-\frac{v_\mathrm{max}^2}{v_\mathrm{min}^2}}\right) \tag{7}$$

where v_min can be a small velocity compared to the normal operational velocity of the vehicle [1].

This representation of the slip ratio and generally the basic interpretations of the terramechanics derivations do not make it possible to directly use the force and torque formulas as constitutive relations in the dynamics formulations. The formulas can be used through a constraint based method where the explicit expressions for the forces and torques are considered together with constraints on the related velocity components. In this way, the effect of the ground reactions are considered through constraints, and the formulas give the interpretations of the set-valued forces associated with the constraints. For example, if we consider that f can represent any of the above force components and v_f is the related velocity then such a constraint relationship with the associated set-valued forces can be given as

$$f = \begin{cases} +\bar{f} & \text{if} \quad v_f^+ \leqslant 0 \\ \in [-\bar{f}, +\bar{f}] & \text{if} \quad v_f^+ = 0 \\ -\bar{f} & \text{if} \quad v_f^+ \geqslant 0 \end{cases} \tag{8}$$

where $\bar{f}$ is the magnitude of such a force and can be interpreted based on terramechanics formulas.

This approach will make it possible to properly model the ground reactions in the dynamics model for a broad range of operating conditions including the cases of zero slip and low wheel velocity. This representation of the wheel-terrain interaction together with the dynamics equations, described by Eqs. (1) and (2), produce a mathematical model in the form of a mixed linear complementarity problem (MLCP), that can be handled with various numerical techniques.

2. Wong-Reece Model

The model proposed by Wong and Reece [2], which is an extension of the model proposed by Bekker [3], characterizes the normal and shear stress distributions at the wheel-terrain contact interface. Therefore, the resultant forces and moments applied to the wheel can be calculated as

$$F_\mathrm{n} = \int_S (\sigma_z + \tau_z)\mathrm{d}S \tag{9}$$

$$F_\mathrm{t} = \int_S \tau_x \mathrm{d}S \tag{10}$$

$$F_\mathrm{c} = \int_S \sigma_x \mathrm{d}S \tag{11}$$

$$T_\mathrm{rr} = \int_S r(\tau - \tau_x)\mathrm{d}S \tag{12}$$

where σ_x and σ_z are the vertical and longitudinal components of the normal stress σ, respectively; and τ_x and τ_z are the vertical and longitudinal components of the shear stress τ, respectively.

The definition of the normal and shear stresses depend on the model used, however a general expression in terms of the angle about the wheel are

$$\sigma = \left(\frac{k_\mathrm{c}}{b} + k_\phi\right) R^n \left(\cos\theta_\mathrm{w} - \cos\theta_1\right) \tag{13}$$

$$\tau = (c + \sigma\tan\phi)\left(1 - \mathrm{e}^{-\frac{j_\mathrm{d}}{K_\mathrm{d}}}\right) \tag{14}$$

where θ_1 is the entrance angle, θ_w is a function of the exit angle θ_2 and the slip ratio, c is the cohesion stress, ϕ is the angle of internal shearing resistance of the terrain, K_d is the shear deformation, and j_d is the shear displacement [4].

3. Cone Index Model

An alternative possibility to characterize traction capabilities is based on the cone index. This has been used in various areas for mobility assessment and modelling [5, 6]. The advantages of this approach include the simplicity of the testing needed to obtain the cone index, and also the availability of databases and information for various different terrains. Here we describe a method of how to include the cone index based representation in the dynamics modelling and simulation framework.

The cone index model estimates the pull generated by a wheel and the applied torque necessary to drive it on different kinds of soil, given steady state conditions. The soil mechanical properties are characterized by the *cone index*, c_i, which is related to the soil strength and is a value obtained empirically using a cone-shaped penetrometer.

The *gross thrust* produced by the vehicle is assumed to be proportional to the torque applied to the wheel in steady state, but here, in the general dynamic case, it is considered as the *traction force*, F_t, coming from the shear stress at the contact interface and applied at the lowest point of the wheel. On the other hand, the *motion resistance* takes into account the effect of the compaction resistance force and the rolling resistant moment, and it is considered as a horizontal force, F_c, applied at the centre of the wheel. Therefore, the rolling resistant moment T_rr is not directly considered in this model.

Therefore, in steady state, the following equation gives the net pull (or *drawbar pull*), F_p, generated by the wheel

$$F_\mathrm{t} = \frac{T_\mathrm{a}}{r} = F_\mathrm{p} + F_\mathrm{c} \tag{15}$$

where T_a is the torque applied to the wheel in steady state, and r is the effective radius of the wheel. In the transient phase and taking into account the inertia effect of the wheel, the resultant force applied to the wheel in the longitudinal direction is

$$F_\mathrm{t} - F_\mathrm{p} - F_\mathrm{c} = m\dot{v}_x \tag{16}$$

and the resultant moment is

$$T_\mathrm{a} - F_\mathrm{t} r = I_y \dot{\omega} \tag{17}$$

where m is the mass of the wheel and I_y is the moment of inertia.

The value of these forces depend on the *mobility number*, a dimensionless parameter, which is defined by Brixius [7] for bias ply tires operating in cohesive-frictional soil as

$$b_{\mathrm{m}} = \left(\frac{c_{\mathrm{i}} D b}{F_{\mathrm{n}}}\right) \frac{1+5\frac{\delta}{H}}{1+3\frac{b}{D}} \tag{18}$$

where $D = 2r$ is the wheel diameter, b is the wheel width, H is the tire section height, and δ is the tire deflection. The gross thrust is defined by the model in terms of the slip ratio, so that the friction coefficient (or *thrust ratio*) is

$$\mu(i_{\mathrm{s}}) = \frac{F_{\mathrm{t}}}{F_{\mathrm{n}}} = 0.88\left(1-\mathrm{e}^{-0.1 b_{\mathrm{m}}}\right)\left(1-\mathrm{e}^{-7.5 i_{\mathrm{s}}}\right)+0.04 \tag{19}$$

and the *motion resistance ratio* is

$$\gamma(i_{\mathrm{s}}) = \frac{F_{\mathrm{c}}}{F_{\mathrm{n}}} = \frac{1.0}{b_{\mathrm{m}}} + \frac{0.5 i_{\mathrm{s}}}{\sqrt{b_{\mathrm{m}}}} + 0.04 \tag{20}$$

3.1 Set-Valued Force Laws

The normal contact force can be defined in terms of the sinkage as

$$F_{\mathrm{n}} = k_{\mathrm{n}} z + c_{\mathrm{n}} \dot{z} \tag{21}$$

where k_{n} and c_{n} are the stiffness and damping coefficients, respectively. Since the cone index model only determines the forces associated with the longitudinal dynamics of the vehicle, the Wong-Reece model can be used for the vertical dynamics, and we thus define the related parameters as proposed in [1]

$$k_{\mathrm{n}} = \frac{F_{\mathrm{n}}^{\mathrm{WR}}}{z} \quad \text{and} \quad c_{\mathrm{n}} = \eta_{\mathrm{n}} k_{\mathrm{n}} \tag{22}$$

where $F_{\mathrm{n}}^{\mathrm{WR}}$ is computed using the Wong-Reece model (see Eq. (9)), and η_{n} is a small positive real number.

Set-valued force laws can be defined for the traction force and resistance force, so that

$$F_{\mathrm{t}}^{+} \begin{cases} = +F_{\mathrm{t}} & \text{if} \quad v_{\mathrm{t}}^{+} \leqslant 0 \\ \in [-F_{\mathrm{t}}, +F_{\mathrm{t}}] & \text{if} \quad v_{\mathrm{t}}^{+} = 0 \\ = -F_{\mathrm{t}} & \text{if} \quad v_{\mathrm{t}}^{+} \geqslant 0 \end{cases} \tag{23}$$

and

$$F_{\mathrm{c}}^{+} \begin{cases} = +F_{\mathrm{c}} & \text{if} \quad v_{x}^{+} \leqslant 0 \\ \in [-F_{\mathrm{c}}, +F_{\mathrm{c}}] & \text{if} \quad v_{x}^{+} = 0 \\ = -F_{\mathrm{c}} & \text{if} \quad v_{x}^{+} \geqslant 0 \end{cases} \tag{24}$$

where $F_{\mathrm{t}} = \mu F_{\mathrm{n}}$ and $F_{\mathrm{c}} = \gamma F_{\mathrm{n}}$ are the traction force and resistance force according to the cone index model.

However, the traction force can be better approximated about the current state using a linear regularization of the constraint force, or *global regularization* (see Fig. 3), which can be defined as

$$F_{\mathrm{t}}^{+} = -c_{\mathrm{t}} v_{\mathrm{t}}^{+} \tag{25}$$

where the viscous damping coefficient

$$c_{\mathrm{t}} = \frac{F_{\mathrm{t}}}{v_{\mathrm{t}}} \tag{26}$$

Moreover, a more detailed approximation can be done via a linearization about the current state, or *local regularization* (see Fig. 3), so that the change of the force is linear with respect to the change in the sliding velocity. Therefore, the force can be expressed as

$$F_{\mathrm{t}}^{+} = F_{\mathrm{t}} - c_{\mathrm{t}}\left(v_{\mathrm{t}}^{+} - v_{\mathrm{t}}\right) \tag{27}$$

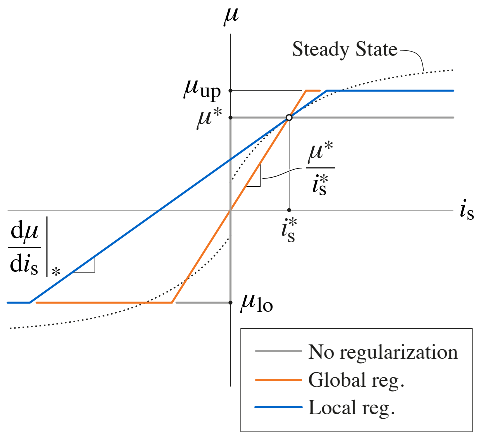

Figure 3: Friction coefficient (or *thrust ratio*) corresponding to the set-valued force law and the possible regularizations

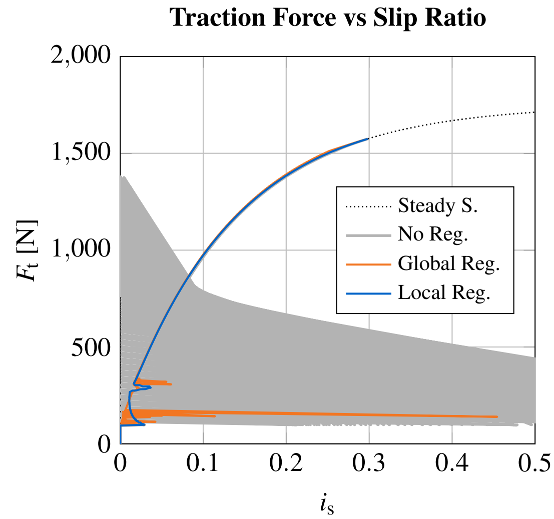

Figure 4: Simulation result of a single wheel model

where c_t is the viscous coefficient that regularizes the force by a linearization of the curve representing the friction coefficient $\mu(i_s)$. Considering that the slip ratio is given as $i_s = -\frac{v_t}{v_{max}}$, a viscous damping coefficient can be defined as

$$c_t = \frac{F_n}{v_{max}} \left.\frac{d\mu}{di_s}\right|_* \tag{28}$$

where $\left.\frac{d\mu}{di_s}\right|_*$ is the slope of the tangent at the current slip ratio value, $\mu^* = \mu(i_s^*)$.

Therefore, the set-valued law for the traction force can be defined as

$$F_t^+ = \begin{cases} F_t^{up} & \text{if} \quad w_t^+ \leqslant 0 \\ F_t - c_t\left(v_t^+ - v_t\right) & \text{if} \quad w_t^+ = 0 \\ F_t^{lo} & \text{if} \quad w_t^+ \geqslant 0 \end{cases} \tag{29}$$

where F_t^{up} and F_t^{lo} are the upper and lower bounds, respectively, which can be taken as $\pm F_t$. The slack variable of the MCLP associated with the friction force bounds $F_t^+ \in [F_t^{lo}, F_t^{up}]$ can be defined as

$$w_t^+ = v_t^+ + \varepsilon_t h F_t^+ - (v_t + \varepsilon_t h F_t) \tag{30}$$

and the regularization term is $\varepsilon_t = \dfrac{1}{c_t h}$, which is generally a small value.

4. Examples

4.1 Single Wheel Test

To verify the novel formulation proposed for the cone index model, a series of simulations have been performed with a quarter vehicle model. The longitudinal dynamics of the vehicle and its behaviour have been tested under some conditions for both transient and steady states. Table 1 shows the system and tire model parameters. First, a driving torque is applied to the wheel, which increases linearly from $T_a = 0$ to $T_a = 600\,\mathrm{Nm}$ for the first 10 s of simulation, and then it remains at $T_a = 600\,\mathrm{Nm}$ for the next 10 s until the system reaches the steady state. An external pull that increases with the velocity is applied to the vehicle, so that a steady state is possible, as can be seen in Fig. 5.

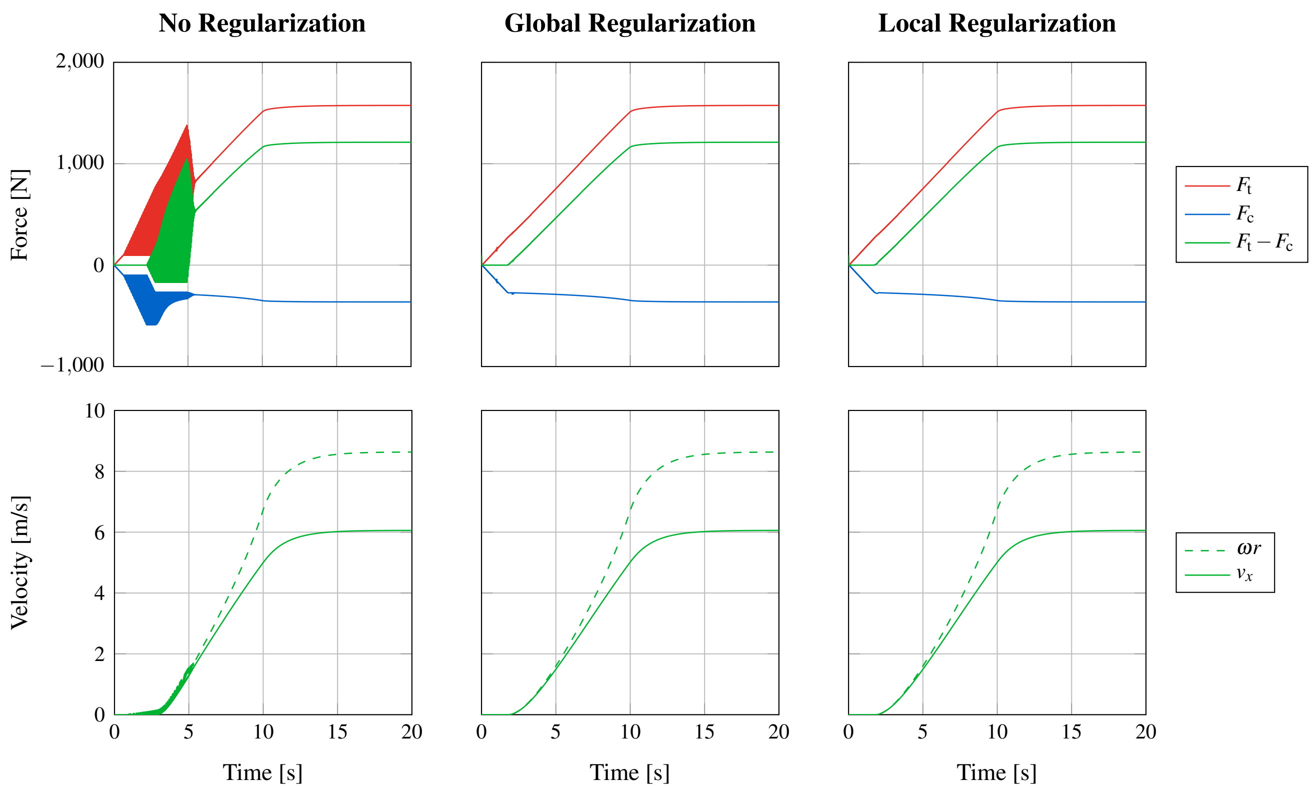

Figure 5: Quarter vehicle simulation results forces (top) and velocities (bottom). Acceleration from rest applying an incremental torque from 0 to 600 Nm in 10 s

Vehicle Properties		
Vehicle Mass	m	1000 kg
Wheel Inertia	I_y	6 kg m^2
Wheel Diameter	D	0.762 m
Wheel Width	b	0.2 m
Section Height	H	0.14 m
Tire Pressure	p	2 bar
Contact Model		
Cone Index	c_i	400 kN/m2

Table 1: Parameters of a quarter vehicle model

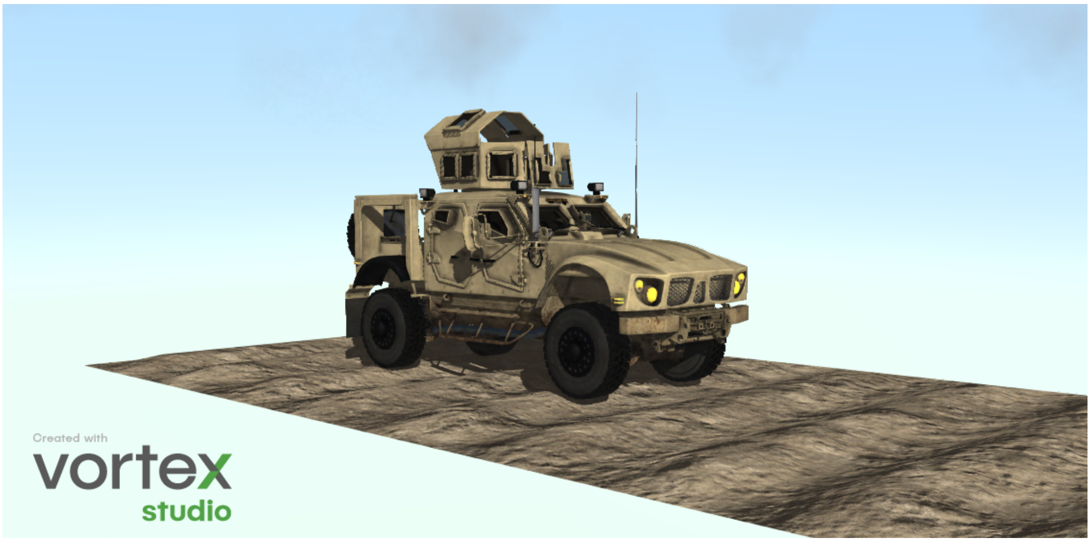

Figure 6: Full vehicle model consisting of 45 rigid bodies

The three different regularized constraint approaches have been used and all of them give similar behaviour. However, the non-regularized model presents some oscillations for small slip ratio compared to the others. This is because the constraint alone is not capable of approximating the steady state curve properly when the traction force increases rapidly, as shown in Fig. 4. Still, it is noteworthy that the simulation reaches the same steady state both with and without the use of regularization. As mentioned above, the cone index model describes the system dynamics in steady state, but it does not capture the full range of dynamic conditions of the wheel-terrain interaction.

4.2 Full Vehicle Test

The simulation of a full vehicle model has been performed in order to verify the tire model under realistic driving conditions (see Fig. 6). The data corresponding to the model with global regularization are shown in Fig. 7. The vehicle has been driven on a flat terrain at full throttle for 10 s with a change of gears after 5 s and reaching a steady state, where the velocity of the vehicle is $v_x = 19.1\,\mathrm{km/h}$. This example shows how the proposed tire model based on the cone index is able to maintain a steady state in complex situations such as this one and also capture the transient phase necessary to reach it.

5. Conclusions

In this paper we discussed elements of a framework that can be used for dynamic simulation of vehicle-terrain systems for a broad range of conditions. It relies on a constraint-based complementarity formulation where the ground reactions are considered together with their complementary kinematic variables to completely define the models of wheel-terrain interactions for a full range of slip. This framework can include various types of wheel-soil and tire models. As a completely novel element, we have shown how soft soil models based on the cone index can be incorporated in dynamic simulation using this modelling framework. Moreover, we also presented numerical results for the simulation of a full vehicle. The proposed approach makes it possible to use these models for challenging operating conditions, such as the transient dynamics during starting, accelerating and braking.

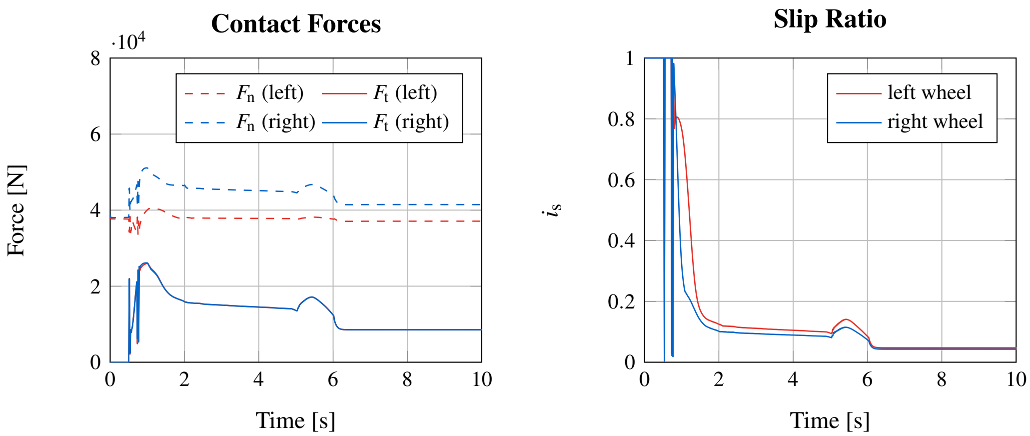

Figure 7: Simulation of the full vehicle model: normal and traction forces (left) and slip ratio (right) of the two driving rear wheels

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