IUTAM Symposium on Co-Simulation and Solver Coupling
September 18 - September 20, 2017, Darmstadt, Germany

Dynamic Interface Modelling for Co-Simulation of Multibody Systems

Albert Peiret, Joe Hewlett, László L. Kovács, József Kövecses

Department of Mechanical Engineering and Centre for Intelligent Machines,
McGill University
845 Sherbrooke Street West, Montréal, Québec H3A 0G4, Canada

Abstract

Computationally efficient simulation of multibody systems is important in many applications. However, some parts of the system, which can be regarded as stiff subsystems, often need to be simulated at a high time-rate to prevent the system from being unstable [1]. These can include cables under high tension, flexible subsystem models and actuators with non-linear dynamics. In this paper, the dynamics of a multibody system is formulated using a co-simulation approach, so that the stiffer subsystem can be simulated at a higher time-rate than the rest of the system.

A subsystem is identified as the *fast* subsystem and the rest of the system forms the *slow* subsystem. The ratio between the corresponding time-steps is assumed to be an integer $N = h_\mathrm{s}/h_\mathrm{f}$, where h_s and h_f are the time-step size of the slow and fast subsystems. The fast subsystem needs to be simulated for N time-steps before the slow subsystem steps forward, and therefore, some assumptions regarding the evolution of the slow subsystem during this time-steps need to be made. The simplest is to assume that the slow subsystem can be considered "stationary" for the duration of the slow time step. While the classical co-simulation approaches rely on the extrapolation of interface variables, such as velocities or forces [1, 2]. Here, a reduced order representation of the slow subsystem will be used to emulate its behaviour during the time-step of the slow subsystem. Similar techinques are also applied in *gluing algorithms*, where subsystem interface quantities are used, e.g., the interface flexibility [3].

Consider that the interface kinematics between the subsystems is described by r equations arranged in array $\mathbf{g}(\mathbf{q}_\mathrm{s}, \mathbf{q}_\mathrm{f})$, where $\mathbf{q}_\mathrm{s}$ and $\mathbf{q}_\mathrm{f}$ contain the generalized coordinates of the slow and fast subsystems. In this case, the interface velocities are

$$\dot{\mathbf{g}} = \mathbf{u}_\mathrm{s} + \mathbf{u}_\mathrm{f} = \mathbf{A}_\mathrm{s}\mathbf{v}_\mathrm{s} + \mathbf{A}_\mathrm{f}\mathbf{v}_\mathrm{f} \tag{1}$$

where $\mathbf{v}_\mathrm{s}$ and $\mathbf{v}_\mathrm{f}$ contain the generalized velocities of the slow and fast subsystems, respectively, and $\mathbf{A}_\mathrm{s}$ and $\mathbf{A}_\mathrm{f}$ are the corresponding Jacobian matrices. Using this partitioning, the dynamic equations of the subsystems can be written as

$$\mathbf{M}_\mathrm{s}\dot{\mathbf{v}}_\mathrm{s} = \mathbf{f}_\mathrm{s} + \mathbf{A}_\mathrm{s}^\mathrm{T}\boldsymbol{\lambda} \tag{2}$$

$$\mathbf{M}_\mathrm{f}\dot{\mathbf{v}}_\mathrm{f} = \mathbf{f}_\mathrm{f} + \mathbf{A}_\mathrm{f}^\mathrm{T}\boldsymbol{\lambda} \tag{3}$$

where $\mathbf{M}_\mathrm{s}(\mathbf{q}_\mathrm{s})$ and $\mathbf{M}_\mathrm{f}(\mathbf{q}_\mathrm{f})$ are the mass matrices of the subsystems, $\mathbf{f}_\mathrm{s}(\mathbf{q}_\mathrm{s}, \mathbf{v}_\mathrm{s}, t)$ and $\mathbf{f}_\mathrm{f}(\mathbf{q}_\mathrm{f}, \mathbf{v}_\mathrm{f}, t)$ are the generalized forces, and $\boldsymbol{\lambda}$ represents the r interface forces.

Let $\mathbf{v}_\mathrm{s}^0$ and $\mathbf{v}_\mathrm{f}^0$ denote the velocities of the subsystems at the beginning of the slow subsystem time-step, and use $\mathbf{v}_\mathrm{s}^1$ and $\mathbf{v}_\mathrm{f}^N$ at the end of the same time-step. Then, a time-stepping scheme can be derived by introducing the finite difference approximation of the accelerations $\dot{\mathbf{v}} = (\mathbf{v}^k - \mathbf{v}^{k-1})/h$, and using constraint relaxation to handle the constraints. With the reduced representation of the slow subsystem parametrized by the interface velocities $\mathbf{u}_\mathrm{s}^k = \mathbf{A}_\mathrm{s}\mathbf{v}_\mathrm{s}^k$, for $k = 1 \ldots N$, equations (2) and (3) yield

$$\begin{bmatrix} \mathbf{M}_\mathrm{f} & -\mathbf{A}_\mathrm{f}^\mathrm{T} \\ \mathbf{A}_\mathrm{f} & h_\mathrm{f}^{-2}\mathbf{C} + \widetilde{\mathbf{M}}_\mathrm{s}^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{v}_\mathrm{f}^k \\ h_\mathrm{f}\boldsymbol{\lambda}^k \end{bmatrix} = \begin{bmatrix} \mathbf{M}_\mathrm{f}\mathbf{v}_\mathrm{f}^{k-1} + h_\mathrm{f}\mathbf{f}_\mathrm{f}^{k-1} \\ -\mathbf{D}h_\mathrm{f}\mathbf{g}^{k-1} - \mathbf{u}_\mathrm{s}^{k-1} - \mathbf{A}_\mathrm{s}\mathbf{M}_\mathrm{s}^{-1}h_\mathrm{f}\mathbf{f}_\mathrm{s}^0 \end{bmatrix} \tag{4}$$

and

$$\mathbf{u}_\mathrm{s}^k = \mathbf{u}_\mathrm{s}^{k-1} + \widetilde{\mathbf{M}}_\mathrm{s}^{-1} h_\mathrm{f} \boldsymbol{\lambda}^k + \mathbf{A}_\mathrm{s} \mathbf{M}_\mathrm{s}^{-1} h_\mathrm{f} \mathbf{f}_\mathrm{s}^0 \tag{5}$$

where $\widetilde{\mathbf{M}}_\mathrm{s} = \left(\mathbf{A}_\mathrm{s} \mathbf{M}_\mathrm{s}^{-1} \mathbf{A}_\mathrm{s}\right)^{-1}$ is the effective mass matrix of the slow subsystem projected to the space of interface velocities collected in $\mathbf{u}_\mathrm{s}$. Quantities with subscript "s" depend on the configuration of the slow subsystem, and they are computed with $\mathbf{q}_\mathrm{s}^0$ during the N time steps of the fast subsystem. Subscript "f" denotes the dependency of certain quantities on the configuration $\mathbf{q}_\mathrm{f}^{k-1}$ of the fast subsystem. The $r \times r$ identity matrix is denoted by $\mathbf{I}$; and $\mathbf{C}$ and $\mathbf{D}$ are the diagonal matrices containing the constraint relaxation and stabilization coefficients $c_i = (k_i + b_i/h_\mathrm{f})^{-1}$ and $d_i = c_i k_i$, for $i = 1 \ldots r$, where k_i and b_i are the interface stiffness and damping, respectively.

Once the N time-steps have been computed for the fast subsystem, the values of the interface force $\boldsymbol{\lambda}^k$, for $k = 1 \ldots N$, can be applied to the slow subsystem using the average $\bar{\boldsymbol{\lambda}} = \frac{1}{N} \sum_{k=1}^{N} \boldsymbol{\lambda}^k$, see Figure 1. This is because the total impulse applied on the fast subsystem is $\boldsymbol{\Lambda}_\mathrm{f} = \sum_{k=1}^{N} h_\mathrm{f} \boldsymbol{\lambda}^k$, and the configuration of the slow one is kept constant along the time-step. Therefore, using that $h_\mathrm{s} = N h_\mathrm{f}$, the impulse applied on the slow subsystem must be given by $\boldsymbol{\Lambda}_\mathrm{s} = h_\mathrm{s} \bar{\boldsymbol{\lambda}}_\mathrm{s}$. Then, the slow subsystem can step forward in time as

$$\mathbf{v}_\mathrm{s}^1 = \mathbf{v}_\mathrm{s}^0 + \mathbf{M}_\mathrm{s}^{-1} h_\mathrm{s} \left(\mathbf{f}_\mathrm{s}^0 + \mathbf{A}_\mathrm{s}^\mathrm{T} \bar{\boldsymbol{\lambda}} \right) \tag{6}$$

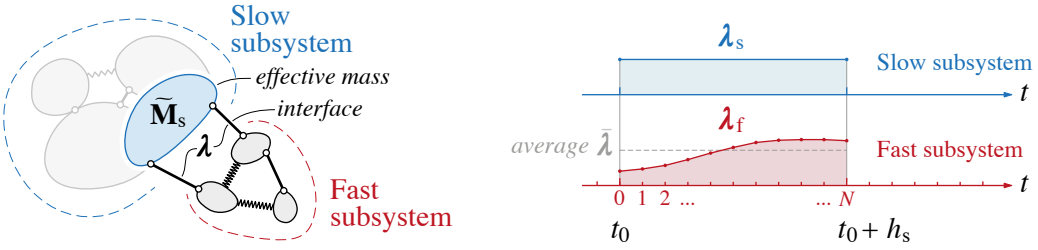

Figure 1: Fast and slow subsystems.

The key idea here, is twofold. First, we have a fast subsystem, and we represent the impulse of the slow subsystem on it by using a reduced model of the slow subsystem, parametrized by the interface velocities $\mathbf{u}_\mathrm{s}$, and the effective mass of the slow subsystem $\widetilde{\mathbf{M}}_\mathrm{s}$. Secondly, the effect of the fast subsystem on the slow subsystem is taken into consideration by applying the average forces exerted by the fast subsystem during the slow time step.

The methodology proposed here has been formulated for multibody systems, with which several examples have been tested. Nevertheless, this idea can be further extended to other fast mechanic or mechatronic systems interacting with the slow multibody system, the effective mass of which can be used to emulate its behaviour at the interface. Moreover, the calculations required in the simulation are reduced by only computing the fast subsystem at a small time-step, and improving performance without compromising much accuracy.

References

[1] Busch, M., and B. Schweizer. "Numerical stability and accuracy of different co-simulation techniques: analytical investigations based on a 2-DOF test model." *Proceedings of The 1st Joint International Conference on Multibody System Dynamics*, 2010.

[2] Arnold, Martin. "Stability of sequential modular time integration methods for coupled multibody system models." *Journal of Computational and Nonlinear Dynamics* 5(3): 031003, 2010.

[3] Wang, J., Ma, Z. D., and Hulbert, G. M. "A gluing algorithm for distributed simulation of multibody systems." *Nonlinear Dynamics* 34: 159–188, 2003.