On the Solvability of Dynamic Formulations for Multibody Systems with Contacts and Coulomb Friction

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***Abstract*— The dynamics of multibody systems with contacts and Coulomb friction is discussed following a general approach. In addition, a new friction cone is introduced, the equation of which is derived and analyzed. This novel geometrical tool makes it possible to assess the solvability of the dynamic equations qualitatively, and also to obtain the values of the friction coefficient that can make the dynamic formulation inconsistent.**

The existence of solution of dynamic formulations for multibody systems with contacts and Coulomb friction, such as robotic systems, may be compromised in some circumstances [1]. Sliding contacts may cause a phenomenon known as *dynamic wedging* (or *dynamic locking*) [2], which is very closely related to the *jamb process* in collisions [3]. If these phenomenon are not taken into account, the formulation may become ill-posed depending on the assumptions made.

The *classic friction cone* is a well-known element of the Coulomb model and can be defined as

$$\kappa_\mu(\boldsymbol{\lambda}_\mathrm{c}) = \boldsymbol{\lambda}_\mathrm{c}^\mathrm{T}\mathbf{Q}_\mu\boldsymbol{\lambda}_\mathrm{c} = \begin{bmatrix}\boldsymbol{\lambda}_\mathrm{t}\\ \lambda_\mathrm{n}\end{bmatrix}^\mathrm{T}\begin{bmatrix}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & -\mu^2\end{bmatrix}\begin{bmatrix}\boldsymbol{\lambda}_\mathrm{t}\\ \lambda_\mathrm{n}\end{bmatrix} = 0 \qquad (1)$$

where $\lambda_\mathrm{n} \geqslant 0$ is the normal force, $\boldsymbol{\lambda}_\mathrm{t} = [\lambda_{\mathrm{t}1}\ \lambda_{\mathrm{t}2}]^\mathrm{T}$ are the friction force components, and $\mathbf{I}$ is the 2×2 identity matrix.

On the other hand, the *generalized friction cone* is a concept that goes beyond the contact forces and takes the system dynamics into account [4]. The shape of this cone depends not only on the friction coefficient, but also on the configuration and mass distribution of the system, and its position can be directly related to the existence of solution of the dynamic formulation [1].

The change in the contact velocity can be expressed as $\mathrm{d}\mathbf{u}_\mathrm{c} = \delta\mathbf{u}_0 + \delta\mathbf{u}_\mathrm{c}$, where the contribution of the contact forces to the change in the contact velocities is

$$\delta\mathbf{u}_\mathrm{c} = \mathbf{A}\mathbf{M}^{-1}\mathbf{A}^\mathrm{T}\boldsymbol{\lambda}_\mathrm{c}\mathrm{d}t = \mathbf{M}_\mathrm{c}^{-1}\boldsymbol{\lambda}_\mathrm{c}\mathrm{d}t \qquad (2)$$

and the contribution of all the other forces acting on the system are represented by $\delta\mathbf{u}_0$. The contact velocity components are defined as $\mathbf{u}_\mathrm{c} = \mathbf{A}\mathbf{v}$, where $\mathbf{v}$ are the n generalized velocities associated with mass matrix $\mathbf{M}$.

Here, a new friction cone is proposed, which is defined in the contact velocity space. In addition, the novel quadratic expression for the change in the contact velocity is derived

$$\kappa_\mathrm{g}(\delta\mathbf{u}_\mathrm{c}) = \delta\mathbf{u}_\mathrm{c}^\mathrm{T}\mathbf{Q}\delta\mathbf{u}_\mathrm{c} = \begin{bmatrix}\delta\mathbf{u}_\mathrm{t}\\ \delta u_\mathrm{n}\end{bmatrix}^\mathrm{T}\begin{bmatrix}\mathbf{Q}_\mathrm{t} & \mathbf{Q}_\mathrm{tn}\\ \mathbf{Q}_\mathrm{tn}^\mathrm{T} & Q_\mathrm{n}\end{bmatrix}\begin{bmatrix}\delta\mathbf{u}_\mathrm{t}\\ \delta u_\mathrm{n}\end{bmatrix} = 0 \qquad (3)$$

where $\mathbf{Q} = \mathbf{M}_\mathrm{c}\mathbf{Q}_\mu\mathbf{M}_\mathrm{c}$. The geometry of this cone can be of interest and help to better understand the dynamics of the system. Its intersection with the plane $\delta u_\mathrm{n} = 0$ is different from a point (the vertex of the cone) if the friction coefficient is greater than a value $\mu \geqslant \mu_\mathrm{jam}$ [3], and in such a case, some contact forces can contribute to the change in the contact velocity with $\delta u_\mathrm{n} < 0$.

Figure 1 illustrates the proposed cone for a rod sliding at 45° on a plane. It shows how its position with respect to the tangent plane affects the system dynamics, which can lead to unrealistic solutions or no solution. For instance, if the contact forces generate a negative change in the normal velocity ($\delta u_\mathrm{n} < 0$), and so do the other forces acting on the system, it is not possible to maintain a unilateral constraint $u_\mathrm{n} \geqslant 0$, and dynamic wedging occurs (see Fig. 1 left). On the other hand, if the contact has to detach but the contact forces contribute with $\delta u_\mathrm{n} < 0$, a constraint could still be applied to keep $u_\mathrm{n} = 0$, which would give a false effect of cohesion (see Fig. 1 right).

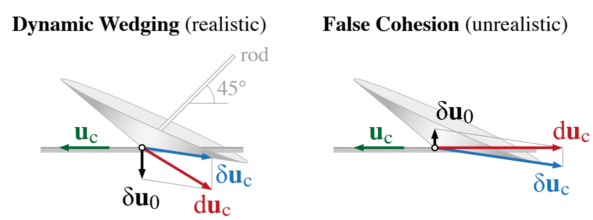

Fig. 1. Possible solutions of a sliding rod at 45° on a plane with $\mu = 2$

In conclusion, this new friction cone is a useful geometric tool that gives more insight into the dynamics of multibody systems with contacts and Coulomb friction. It is able to capture phenomena such as dynamic wedging and it can be used to assess the solvability of the dynamic formulation when unilateral constraints are used to model contact.

References

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