Interface Models in Co-Simulation of Nonsmooth Systems

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EXTENDED ABSTRACT

1 Introduction

Modelling and simulating systems with components of different physical nature and time-scale can be challenging, especially if good simulation performance is important [1]. Co-simulation allows for coupling numerical simulations of different subsystems by exchanging interface variables at discrete communication time-points, which makes this approach modular and convenient for developing complex models. However, coupling subsystems though interface variables can affect simulation stability due to discontinuities and time delay of these variables. Although iterative co-simulation schemes exhibit good numerical stability [2], they can be prohibitive in real-time applications due to the high computational cost. On the other hand, extrapolation techniques avoid the need for iterations by predicting the input variables between communication updates, which translates into better performance [3]. However, using previous values of the interface variables to extrapolate inputs can give wrong predictions, especially in nonsmooth systems such as those with unilateral contact.

Interface models represent a physics-based prediction of the interface variables. Here, an *interface model* (IM) for nonsmooth multibody systems is proposed, which can include unilateral contact and friction. The interaction between the elements in the system is represented by constraints, and the dynamics of the model at the interface are characterized by effective mass and force terms. This models can be used in multirate co-simulation setups, where multibody systems are coupled with subsystems simulated at higher time-rates, and IM can provide a prediction of the multibody system between communication points. Additionally, a model of a hydraulic crane is used to show that larger time-step sizes can be used with the proposed IM, allowing for a reduction in computational time.

2 Dynamics of Nonsmooth Multibody Systems

The p generalized coordinates and the n generalized velocities of the multibody system can be arranged in the arrays $\mathbf{q}$ and $\mathbf{v}$, respectively. In general, $p \geq n$, depending on the chosen parametrizations. The interaction between the bodies can be parametrized by a set of n_c velocity components arranged in the array $\mathbf{w}_c$, which can be related to the generalized velocities as

$$\mathbf{w}_c = \mathbf{A}\mathbf{v} \tag{1}$$

where $\mathbf{A}(\mathbf{q})$ is the $n_c \times n$ constraint Jacobian matrix. Such interactions can be modelled via kinematic constraints, either bilateral ($\mathbf{w}_c = \mathbf{0}$) or unilateral ($\mathbf{w}_c \geq \mathbf{0}$). The dynamic equations of a multibody system can be written as

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{c} = \mathbf{f}_a + \mathbf{f}_i + \mathbf{A}^T \boldsymbol{\lambda}_c \tag{2}$$

where $\mathbf{M} = \mathbf{M}(\mathbf{q})$ is the $n \times n$ mass matrix, $\mathbf{c}$ is the $n \times 1$ array of Coriolis and centrifugal terms, and $\mathbf{f}$ is the $n \times 1$ array of generalized forces. The array $\boldsymbol{\lambda}_c$ contains the n_c constraint reactions, which can be forces or moments depending on the kind of velocity component in $\mathbf{w}_c$ is constrained. The generalized applied forces $\mathbf{f}_a$ are known, and the interface forces $\mathbf{f}_i$ are exchanged with the other co-simulated subsystems.

The dynamics of nonsmooth systems is usually formulated at the impulse–momentum level for each time-step. In general, the formulation for multibody systems with bilateral and unliateral constraints can be written as a *mixed linear complementarity problem* (MLCP)

$$\left.\begin{aligned} &\begin{bmatrix} \mathbf{M} & -\mathbf{A}^T \\ \mathbf{A} & \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{v}^+ \\ h\boldsymbol{\lambda}_c^+ \end{bmatrix} + \begin{bmatrix} \mathbf{p} \\ \boldsymbol{\phi}/h \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{w}_c^+ \end{bmatrix} \\ &\mathbf{w}_c^+ \quad \text{compl. to} \quad \boldsymbol{\lambda}_c^+ \in \left[\boldsymbol{\lambda}_c^{low}, \boldsymbol{\lambda}_c^{upp}\right] \end{aligned}\right\} \tag{3}$$

where $\mathbf{v}^+$ contains the unknown generalized velocities at the end of the time-step, h is the step size, $\mathbf{p} = \mathbf{M}\mathbf{v} + h(\mathbf{f}_a + \mathbf{f}_i - \mathbf{c})$ is known, $\boldsymbol{\lambda}_c^{low}$ and $\boldsymbol{\lambda}_c^{upp}$ are the lower and upper force bounds respectively. The matrix $\mathbf{C}$ is a $r \times r$ positive semi-definite diagonal matrix with regularization terms if constraint regularization is used to cope with redundancy, and $\boldsymbol{\phi}$ accounts for the constraint violation in such a case. This force bounds are enforced through complementarity, so that the constraint velocity is defined to be positive ($\mathbf{w}_c \geq \mathbf{0}$) if the force reaches the lower bound, or negative ($\mathbf{w}_c \leq \mathbf{0}$) if the force reaches the upper bound, and zero ($\mathbf{w}_c = \mathbf{0}$) otherwise. This complementarity condition can be used to define both bilateral and unilateral constraints. Moreover, friction at the contact interface can also be described with such conditions by using faceted approximations of the Coulomb model.

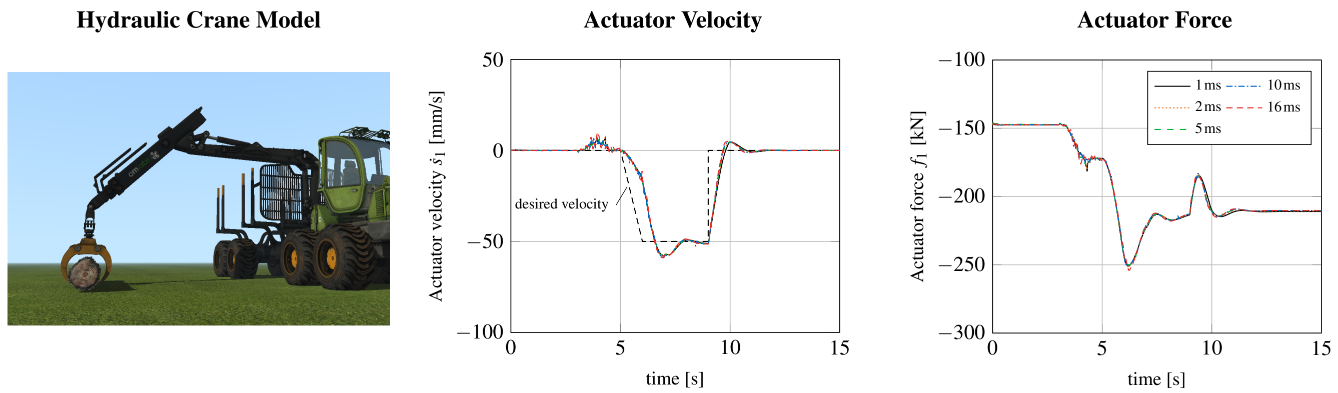

Figure 1: Numerical experiments with a co-simulation setup of a hydraulic crane using an interface model.

3 Interface Models in Multibody Systems

The interface of the multibody system can be parametrized with the r velocity components $\mathbf{w}_{\mathrm{i}} = \mathbf{D}\mathbf{v}$, where $\mathbf{D}(\mathbf{q})$ is the $r \times n$ interface Jacobian matrix. Then, the r interface force components can be arranged in the array $\boldsymbol{\lambda}_{\mathrm{i}}$, which are used to define the generalized interface force as $\mathbf{f}_{\mathrm{i}} = \mathbf{D}^{\mathrm{T}}\boldsymbol{\lambda}_{\mathrm{i}}$. Using the parametrization given by interface velocity components $\mathbf{w}_{\mathrm{i}}$, the dynamic equations of the IM can be written as

$$\widetilde{\mathbf{M}}_{\mathrm{i}}\dot{\mathbf{w}}_{\mathrm{i}} = \widetilde{\boldsymbol{\lambda}}_{\mathrm{i}} + \boldsymbol{\lambda}_{\mathrm{i}} \tag{4}$$

where $\widetilde{\mathbf{M}}_{\mathrm{i}}$ is the *effective mass* matrix of the system at the interface, and $\widetilde{\boldsymbol{\lambda}}_{\mathrm{i}}$ is the *effective force* that takes into account the applied forces $\mathbf{f}_{\mathrm{a}}$ and other Coriolis and centrifugal terms. However, the effective mass and force terms can change quite significantly in non-smooth systems due to contact detachment and stick-slip transitions. Therefore, we define them as

$$\widetilde{\mathbf{M}}_{\mathrm{i}} = \left(\mathbf{D}(\mathbf{I}-\mathbf{P}_{\mathrm{c}})\mathbf{M}^{-1}\mathbf{D}^{\mathrm{T}}\right)^{-1} \qquad \text{and} \qquad \widetilde{\boldsymbol{\lambda}}_{\mathrm{i}} = \widetilde{\mathbf{M}}_{\mathrm{i}}\mathbf{D}\Big((\mathbf{I}-\mathbf{P}_{\mathrm{c}})\mathbf{M}^{-1}\big(\mathbf{f}_{\mathrm{a}} - \mathbf{c} + \mathbf{A}_{\tau}^{\mathrm{T}}\boldsymbol{\lambda}_{\tau}^{+}\big) - \mathbf{P}_{\mathrm{c}}\mathbf{v}h^{-1} - \widetilde{\boldsymbol{\phi}}_{\alpha}h^{-2}\Big) \tag{5}$$

where $\mathbf{P}_{\mathrm{c}} = \mathbf{M}^{-1}\mathbf{A}_{\alpha}^{\mathrm{T}}\left(\mathbf{A}_{\alpha}\mathbf{M}^{-1}\mathbf{A}_{\alpha}^{\mathrm{T}} + \mathbf{C}_{\alpha}\right)^{-1}\mathbf{A}_{\alpha}$ is the $n \times n$ projection matrix that uses only the *active contraints* $\mathbf{w}_{\alpha} = \mathbf{A}_{\alpha}\mathbf{v}$, i.e., with the force withing bounds $\boldsymbol{\lambda}_{\mathrm{c}}^{+} \in \left(\boldsymbol{\lambda}_{\mathrm{c}}^{\mathrm{low}}, \boldsymbol{\lambda}_{\mathrm{c}}^{\mathrm{upp}}\right)$, and $\widetilde{\boldsymbol{\phi}}_{\alpha} = \mathbf{M}^{-1}\mathbf{A}_{\alpha}^{\mathrm{T}}\left(\mathbf{A}_{\alpha}\mathbf{M}^{-1}\mathbf{A}_{\alpha}^{\mathrm{T}} + \mathbf{C}_{\alpha}\right)^{-1}\boldsymbol{\phi}_{\alpha}$. The other constraints with the force at the bound (or *tight constraints* $\mathbf{w}_{\tau} = \mathbf{A}_{\tau}\mathbf{v}$) are considered as applied forces in $\widetilde{\boldsymbol{\lambda}}_{\mathrm{i}}$ and do not affect the effective mass, which is the case of detaching contact and kinetic friction.

4 Example

The model of a crane with two hydraulic actuators and a log gripper was used to illustrate the performance of the proposed IM in a co-simulation setup. A total of 18 bodies and 22 joints constitute the model, which include spherical, revolute, and prismatic joints. The maneuver consists in grasping a log of 500 kg and lift it up to a certain height. For this, a desired velocity was provided as an input of the PD controller of the valve displacement of the hydraulic system. The time-step size of the hydraulic system was 0.2 ms in all the numerical experiments, whereas several values were used for the time-step size of the multibody system, and the communication step size (or macro time-step) was set equal to the one of the multibody system. Figure 1 shows the results of the numerical experiments using the IM. The actuator velocity and force of the first actuators are shown for different time-step sizes of the multibody system. The step size was increased up to 16 ms without loosing stability, which makes the system suitable for simulation at interactive rates.

5 Conclusions

Keeping numerical stability in co-simulation setups can be challenging, especially if non-iterative schemes are used. Interface models (IMs) improve stability and allow for using large step sizes, which makes the simulation suitable for real-time applications. Moreover, the proposed formulation for IMs of nonsmooth multibody systems can include unilateral contact and friction.

References

[1] Rahikainen, J., Kiani, M., Sopanen, J., Jalali, P., Mikkola, A., "Computationally efficient approach for simulation of multibody and hydraulic dynamics," *Mechanism and Machine Theory* 130 (2018) 435–446.

[2] Schweizer, B., Pu L., and Daixing L., "Explicit and implicit cosimulation methods: Stability and convergence analysis for different solver coupling approaches," *Journal of Computational and Nonlinear Dynamics* 10.5 (2015): 051007.

[3] Oberschelp, O., and Vocking, H. . "Multirate simulation of mechatronic systems," *Proceedings of the IEEE International Conference on Mechatronics* (2004).